Appendix

SEIR-Net model description

The SEIR-Net Model is an extension of the SEIR epidemiological model. With the population partitioned into \( n \) groups, the SEIR-Net model consists of a system \( 4n \) ordinary differential equations and a set of initial conditions. The system solution consists of \( 4n \) time functions in days, which show the evolution of the epidemic's variables (number of susceptible, exposed, infected (reported and unreported) and removed over time.

SEIR-Net Model Parameters

The following variables and parameters are used in the SEIR-Net Model:

- \( N_{total} \) = Total number of inhabitants (\( N = 2500000 \) for Belo Horizonte)
- \( ng \) = number of groups (4 in BH)
- \( \text{fracg} \) = vector of the population fractions of each of the \( ng \) groups
  ((0.1170,0.2176,0.5233,0.1421) in BH)
- \( N \) = vector of the total number of individuals (\( N = N_{total} \times \text{fracg} \))
- \( S \) = vector of the total number of susceptible
- \( E \) = vector of the total number of exposed
- \( Ir \) = vector of reported number of infected
- \( In \) = vector of the number of unreported infected
- \( I \) = vector of the total number of infected (reported or not)
- \( R \) = vector of the total number of removed

(Although \( S, E, I \) and \( R \) vary, the \( N = S + E + I + R \) ratio is always valid.)
- \( \mu \) = reducing factor for the transmission rate of the unreported infected.
  (Here \( \mu = 1 \) was used)
- \( F \) = Contact Fraction Matrix, where the entry \( F_{ij} \) indicates the contact intensity of virus transmission from an individual in the group \( i \) to an individual in the group \( j \)
- \( B \) = parameter of virus transmission to individuals in distancing
  (\( B = 1,226 \), obtained by adjusting least squares for Belo Horizonte data)
- \( \alpha \) = Proportion of infected people who will be registered as reported cases
  (\( \alpha = 0.05 \): there are 20 times more current cases than reported cases)
- \( Z \) = Average incubation period (\( Z = 3.69 \) days, according to [Li2020])
- \( Dr \) = vector of average duration of the infectious period in reported cases
- \( Dn \) = vector of average duration of the infectious period in unreported cases
  (\( Dr \) and \( Dn \) use values of 3.48 days, according to [Li2020])
- \( n_{max} \) = duration in days of the simulation

SEIR-Net System of Differential Equations

The SEIR-Net model is governed by a system of \( 4 \times ng \) differential equations. To exemplify, we will explicitly show the equations for four groups in the population (\( ng = 4 \)) with an interaction network between the 4 age groups using the 4x4 matrix \( F = [F_{ij}] \), where \( F_{ij} \) is the social contact factor.
that measures contact intensity between an individual in group $i$ who transmits the virus to an individual in group $j$. The symbol $(\cdot)'$ indicates derivative in relation to time.

$$I[1] = Ir[1] + \mu*In[1]$$


$$(Ir[1])' = \alpha*E[1]/Z - Ir[1]/Dr[1]$$
$$(Ir[2])' = \alpha*E[2]/Z - Ir[2]/Dr[2]$$
$$(Ir[3])' = \alpha*E[3]/Z - Ir[3]/Dr[3]$$
$$(Ir[4])' = \alpha*E[4]/Z - Ir[4]/Dr[4]$$

$$(In[1])' = (1-\alpha)*E[1]/Z - In[1]/Dn[1]$$
$$(In[2])' = (1-\alpha)*E[2]/Z - In[2]/Dn[2]$$
$$(In[3])' = (1-\alpha)*E[3]/Z - In[3]/Dn[3]$$
$$(In[4])' = (1-\alpha)*E[4]/Z - In[4]/Dn[4]$$

**Initial conditions**

The system described above is initialized with 1 exposed individual, divided proportionally between the $ng$ groups according to their proportions in the population:

- $E = \frac{g}{S}$
- $S = N - \frac{g}{S}$
- $I = [0, ..., 0]$
- $R = [0, ..., 0]$
- $h = 1.0$ (one day step)
- $nmax = 120$ days

**Numerical Solution**

The SEIR-Net System is solved numerically with the Fourth-order Runge-Kutta method.

**Program in R Language**

The SEIR-Net method was implemented in the R language and can be downloaded from the following link.

https://drive.google.com/open?id=1Dff5IUcOh3q1uh02izhY5jSK8Ocy5f4P