Appendix

SEIR-Net model description

The SEIR-Net Model is an extension of the SEIR epidemiological model. With the population partitioned into *n* groups, the SEIR-Net model consists of a system 4*n* ordinary differential equations and a set of initial conditions. The system solution consists of 4*n* time functions in days, which show the evolution of the epidemic's variables (number of susceptible, exposed, infected (reported and unreported) and removed over time.

SEIR-Net Model Parameters

The following variables and parameters are used in the SEIR-Net Model:

Ntotal = Total number of inhabitants (N = 2500000 for Belo Horizonte)
ng = number of groups (4 in BH)
fracg = vector of the population fractions of each of the ng groups
((0.1170,0.2176,0.5233,0.1421) in BH)
N = vector of the total number of individuals (N = Ntotal * fracg)
S = vector of the total number of susceptible
E = vector of the total number of exposed
Ir = vector of reported number of infected
In = vector of the number of unreported infected
I = vector of the total number of infected (reported or not)
R = vector of the total number of removed
(Although S, E, I and R vary, the N = S + E + I + R ratio is always valid.)
mu = reducing factor for the transmission rate of the unreported infected.
(Here mu = 1 was used)
F = Contact Fraction Matrix, where the entry F_{ij} indicates the contact intensity of virus transmission from an individual in the group i to an individual in the group j
B = parameter of virus transmission to individuals in distancing
(B = 1,226, obtained by adjusting least squares for Belo Horizonte data)
alpha = Proportion of infected people who will be registered as reported cases
(alpha = 0.05: there are 20 times more current cases than reported cases)
Z = Average incubation period (Z = 3.69 days, according to [Li2020])
Dr = vector of average duration of the infectious period in reported cases
Dn = vector of Average duration of the infectious period in unreported cases
(Dr and Dn use values of 3.48 days, according to [Li2020])
nmax = duration in days of the simulation

SEIR-Net System of Differential Equations

The SEIR-Net model is governed by a system of 4**ng* differential equations. To exemplify, we will explicitly show the equations for four groups in the population (*ng* = 4) with an interaction network between the 4 age groups using the 4x4 matrix F = [F [i, j]], where F [i, j] is the social contact factor that measures contact intensity between an individual in group *i* who transmits the virus to an individual in group *j*. The symbol ()'indicates derivative in relation to time.

I[1] = Ir[1] + mu*In[1]
I[2] = Ir[2] + mu*In[2]
I[3] = Ir[3] + mu*In[3]
I[4] = Ir[4] + mu*In[4]
(S[1])'=-(F[1,1]*I[1] +F[2,1]*I[2] +F[3,1]*I[3] +F[4,1]*I[4])*(B/Ntotal)*(1-I[1]/Ntotal)*S[1]
(S[2])'=-(F[1,2]*I[1] +F[2,2]*I[2] +F[3,2]*I[3] +F[4,2]*I[4])*(B/Ntotal)*(1-I[2]/Ntotal)*S[2]
(S[3])'=-(F[1,3]*I[1] +F[2,3]*I[2] +F[3,3]*I[3] +F[4,3]*I[4])*(B/Ntotal)*(1-I[3]/Ntotal)*S[3]
(S[4])'=-(F[1,4]*I[1] +F[2,4]*I[2] +F[3,4]*I[3] +F[4,4]*I[4])*(B/Ntotal)*(1-I[4]/Ntotal)*S[4]
(E[1])'= (F[1,1]*I[1]+F[2,1]*I[2]+F[3,1]*I[3]+F[4,1]*I[4])*(B/Ntotal)*(1-I[1]/Ntotal)*S[1]-E[1]/Z
(E[2])'= (F[1,2]*I[1]+F[2,2]*I[2]+F[3,2]*I[3]+F[4,2]*I[4])*(B/Ntotal)*(1-I[2]/Ntotal)*S[2]-E[2]/Z
(E[3])'= (F[1,3]*I[1]+F[2,3]*I[2]+F[3,3]*I[3]+F[4,3]*I[4])*(B/Ntotal)*(1-I[3]/Ntotal)*S[3]-E[3]/Z
(E[4])'= (F[1,4]*I[1]+F[2,4]*I[2]+F[3,4]*I[3]+F[4,4]*I[4])*(B/Ntotal)*(1-I[4]/Ntotal)*S[4]-E[4]/Z
(Ir[1])'= alpha*E[1]/Z - Ir[1]/Dr[1]
(Ir[2])'= alpha*E[2]/Z - Ir[2]/Dr[2]
(Ir[3])'= alpha*E[3]/Z - Ir[3]/Dr[3]
(Ir[4])'= alpha*E[4]/Z - Ir[4]/Dr[4]
(In[1])'= (1-alpha)*E[1]/Z - In[1]/Dn[1]
(In[2])'= (1-alpha)*E[2]/Z - In[2]/Dn[2]
(In[3])'= (1-alpha)*E[3]/Z - In[3]/Dn[3]
(In[4])'= (1-alpha)*E[4]/Z - In[4]/Dn[4]

Initial conditions

The system described above is initialized with 1 exposed individual, divided proportionally between the *ng* groups according to their proportions in the population:

E = fracg
S = N-fracg
I = [0, ..., 0]
R = [0, ..., 0]
h = 1.0 (one day step)
nmax = 120 days

Numerical Solution

The SEIR-Net System is solved numerically with the Fourth-order Runge-Kutta method.

Program in R Language

The SEIR-Net method was implemented in the R language and can be downloaded from the following link.

https://drive.google.com/open?id=1Dff5IUcOh3q1uh02izhY5jSK8Ocy5f4P